Coherent Phase Wide Band Demodulation Technique for Turbomachinery Cavitation Detection and Monitoring

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ABSTRACT: This paper discusses a Coherent Phase Wide Band Demodulation (CPWBD) technique for turbopump cavitation detection and monitoring. The principle of cavitation detection is based on the unique nonlinear modulation phenomenon associated with pump cavitation in which the periodic shaft rotational motion (and/or its harmonic motion) amplitude modulates the wide-band noise generated from the collapse of cavitation bubbles. However, these periodicities associated with shaft rotation are hidden during the physical wide band modulation (WBM) process and cannot be identified within the conventional power spectral density (PSD) of the monitored dynamic signal. Such periodicity, hidden well within wide band noise, thus provides a unique signature and serves as the basis for effective cavitation detection. Existing techniques for cavitation detection utilize envelope analysis or full-wave rectification (FWR) spectral analysis to demodulate dynamic measurement signals over an isolated band of high frequency wide band noise in order to recover hidden cavitation-generated periodicity However, the conventional FWR technique is subject to a practical limitation in that any discrete components present in the high frequency regions of interest, due to other vibration sources (mechanical, structural, etc.), will erroneously generate discrete peaks in the resulting demodulated signal appearing as genuine recovered hidden periodicities. The CPWBD technique provides an effective alternative to avoid such discrete interference. The method is based on an inherent signal property associated with the WBM signal, where a unique coherent phase relationship exists among all interacting spectral components associated with the cavitation process. The CPWBD technique can thus identify hidden periodicity by searching for such a coherent phase relationship. An inherent characteristic of the CPWBD is that it is not affected by any linearly superimposed discrete components, and as a result, overcomes the limitation due to discrete interference. Application examples with Space Shuttle Main Engine (SSME) turbopump test data are utilized to demonstrate the effectiveness of the CPWBD for cavitation detection.

KEY WORDS: Cavitation Detection, Demodulation, Impeller, Inducer, Modulation, Turbopump

INTRODUCTION: Cavitation within turbopumps can degrade pump performance and cause excessive vibration along with pump blade pitting. This phenomenon occurs when local fluid pressures in the accelerated flow areas around and within rotating pump blades drop below the vapor pressure. The damaging effects of cavitation are associated with the pressure shock waves generated by the collapse of the vapor bubbles, which impact the adjacent metal surfaces to cause pitting and erosion. The collapse of cavitation bubbles also generates broad band noise in the surrounding fluid and structure, with noise levels proportional to intensity of the cavitation. The principle for cavitation detection is based on the nonlinear modulation phenomenon between the bubble collapse wide-band noise with some synchronous (RPM or harmonics) and/or non-synchronous periodic components. However, the periodicities modulating with the wide band noise become hidden in the resulting WBM signal and cannot be identified with the conventional power spectral density (PSD) function. Such hidden periodicity thus provides a unique signature in the wide band noise conducive to cavitation detection.

Existing techniques for cavitation detection utilize envelope analysis or full-wave rectification (FWR) spectral analysis [1] to demodulate the dynamic measurement signals over an isolated band of high frequency wide band noise in order to recover any cavitation-generated hidden periodicity. The magnitude of the recovered hidden periodicity is then used to quantify cavitation intensities; while the frequency of the recovered hidden periodicity is used to identify various types of cavitation under different pumping conditions. Other experimental work has suggested that [2, 3] cavitation erosion results from the collapse of swirling transient vortices, originating at the leading edge of the hydrofoil. These vortices were found to shed periodically according to a constant Strouhal number. Also observed was that when erosion levels were high a strong modulating frequency was recovered at the periodic vortex shedding rate.

Experimental study of cavitation in hydroturbines [4] indicates that the recovered modulating frequency is found to be the rotating turbine blade/wicket-gate passing frequency when cavitation develops on all of the turbine blades (full blade cavitation). A recent Alternate Turbopump Development (ATD) High Pressure Oxygen Turbopump (HPOTP) inducer flow test series using water conducted at NASA/MSFC to investigate cavitation-induced vibration [5, 6] observed a fairly repeatable sequence of cavitation. At relatively high inlet pressure, small cavitation clouds are present behind each of the four inducer blades (full blade cavitation). The second type of cavitation occurred when the inlet pressure was lowered until two symmetric cavitation clouds appeared behind alternate blades (alternate blade cavitation). A third type of cavitation occurred just before head fall-off when two alternate blade cavitation clouds of different size formed in the inducer. In each case, distinct pressure oscillation frequencies are associated with each type of cavitation. Dominant hidden periodicities recovered from the demodulated signal were found to be 4N (4th harmonic of RPM), 2N and 1N frequencies respectively for each of these three cavitation types [7, 8]. Another recent SSME ATD (High Pressure Fuel Turbopump) HPFTP impeller flow test series at NASA/MSFC produced cavitation-induced oscillations at synchronous frequency multiples, as well as at non-synchronous anomalous frequencies, as narrow band random signals, and as broadband random noise. Furthermore, modulation between the non-synchronous components and the synchronous frequency multiples, which generated sideband signatures, was also observed.

These experimental observations reported by various researchers indicate that cavitation-generated modulation phenomenon provides a unique signature to monitor the cavitation process within a wide variety of pumping systems. In addition, the detection of modulating periodic frequency or hidden periodicity within the wide band modulation signal provides a way to identity the type of cavitation under various pumping conditions. Recovery via demodulation of such hidden periodicity thus forms the basis of signal processing for cavitation detection and monitoring. Envelope or full-wave rectification spectral analysis is subject to a practical limitation in that any discrete components present in the high frequency regions of interest, due to other vibration sources (mechanical, structural, etc.), will erroneously generate discrete peaks in the resulting demodulated signal appearing as genuine recovered hidden periodicities. This limitation is critical in assessing the performance of rocket engines under severe operational environments in which other vibration sources contribute many high frequency components which can corrupt the demodulation signal.

The Coherent Phase Wide Band Demodulation method discussed in this paper provides an alternative for wide band demodulation which can avoid such discrete interference. The CPWBD method is based on the signal property within an ordinary (linear) wide band noise signal, spectral components at different frequencies are statistically independent. However, within a signal composed of wide band modulation (WBM) noise coupled with a hidden periodicity, a unique coherent phase relationship exists among all interacting components. The CPWBD technique can thus identify hidden periodicity by searching for such a coherent phase relationship. An inherent characteristic of the CPWBD is that it is not affected by any linearly superimposed discrete components since only phase information is utilized in searching for cavitation signatures. As a result, CPWBD provides an effective way to overcome the critical limitation of discrete interference suffered by the conventional FWR method. The CPWBD technique utilizes a Phase-Only (PO) filter coupled with an envelope detector to search for the unique cavitation-generated coherent phase relationship. Therefore, the CPWBD method offers the unique ability to accurately detect the cavitation condition without being subject to high frequency discrete interference. Application examples with the Space Shuttle Main Engine (SSME) turbopump test data are utilized to demonstrate the effectiveness of the CPWBD for cavitation detection.

WIDE-BAND MODULATION (WBM) SIGNAL: The signal processing techniques, upon which effective cavitation detection and monitoring is based, rely a unique phenomenon associated with cavitation physics:

When cavitation occurs in a rotating system, some periodic components related to the shaft rotational motion will amplitude modulate the wide-band noise generated from the collapse of cavitation bubbles.

A simplified signal model for a Wide-Band Modulation signal can be formulated as a periodic wave multiplied by a wide-band noise signal N(t):

$$x(t) = [\ 1 + r\cos(\omega_r t)]\ N(t) \tag{1}$$

where $\cos(\omega_r t)$ represents some periodic motion associated with shaft rotational motion, and N(t) is zero-mean Gaussian White or Color Noise. A special kind of periodicity exists in such a WBM signal but is well hidden. An ordinary power spectral density (PSD) will not show a discrete peak at the frequency of the periodic component. This can be easily deduced from studying its signal model. The operation between N(t) and $\cos(\omega_r t)$ in the time domain is multiplication, but this multiplication becomes a convolution in the frequency domain. Since the PSD of noise is flat and that of a sine wave is a delta function, the convolution of these two PSD functions remains flat without any discrete peak. For this reason, a conventional PSD is unable to identify periodic components hidden within a Wide-Band Modulation signal.

FULL-WAVE RECTIFICATION WIDE-BAND DEMODULATION: The Full-Wave Rectification Wide-Band Demodulation (FWRWBD) technique for demodulating a WBM signal can be better understood from the tri-spectral approach. Dwyer [9] proposed a special tri-spectral function to identify the existence of hidden periodicity in underwater sonar signal processing. This tri-spectrum is based on the Fourier Transform of a special tri-correlation function $R_{xxxx}(\tau)$ of a measured signal x(t), which is defined as:

$$\begin{aligned} R_{xxxx}(\tau) &= E[\ x(t)\ x(t)\ x(t+\tau)\ x(t+\tau)\ ] \\ &= E[\ x^2(t)\ x^2(t+\tau)\ ] = R_{yy}(\tau) \end{aligned} \tag{2}$$

Where $y(t) = x^2(t)$

The special tri-spectrum $T(\omega)$ is defined as the Fourier Transform of $R_{xxxx}(\tau)$. Notice that, the function $R_{xxxx}(\tau)$ reduces to the ordinary auto-correlation of y(t), where y(t) is simply the square of the original signal x(t). In other words, this special tri-spectrum is equal to the ordinary PSD of the square of the original signal. By examining the WBM signal model, it can be easily seen why such a simple squaring operation can recover the hidden periodicity in a WBM signal. The square of x(t) can be written as:

$$
\begin{aligned}
x(t)^2 &= [\, 1+ r \operatorname{Cos}(\omega_r t) \,]^2 \; [\, N(t) \,]^2 \\
&= [\, 1.5 + 2r \operatorname{Cos}(\omega_r t) + 0.5\, r^2 \operatorname{Cos}(2\, \omega_r t) \,] \, [DC + N'(t) \,]
\end{aligned} \tag{3}
$$

Where N'(t) is defined by the following relationship:

$$
[\, N(t) \,]^2 = [DC + N'(t) \,] \tag{4}
$$

When squaring a zero-mean noise signal, an additional bias, or DC (mean value) component is introduced, and this DC component is multiplied by the periodic component Cos(ωrt). The resulting component then becomes superpositioned on the new noise component N'(t). It is this new DC-introduced superposition term that allows the recovery of the periodic component hidden in the original WBM signal. To maintain the dynamic range of the demodulated signal, the FWRWBD method replaces the squaring operation by an absolute value operation since both operators are effective in recovering the hidden periodicity.

CAVITATION GENERATED WBM SIGNAL: The signal generated by cavitation can be modelled as the multiplication of two separate components p(t) and N(t):

$$
x(t) = p(t)\, N(t) \tag{5}
$$

Here, N(t) represents the wide-band high frequency noise generated from the collapse of cavitation bubbles, while p(t) represents some periodic pressure fluctuation associated with the shaft rotational motion. This pressure signal p(t) contains a DC component due to its static pressure component P_{static},

$$
p(t) = [\, \underset{(DC)}{P_{static}} + P(t)dynamic \,]. \tag{6}
$$

Therefore, a cavitation generated pressure signal is a typical WBM signal and is a good candidate for WBD processing. However, in most operational environments, the spectra in the low frequency region always contains the fundamental rotor synchronous (Sync) frequency component along with its harmonics (2N, 3N, 4N...), which are generated from the combination of all the effects of rotor dynamics, structural dynamics and hydrodynamics. These combined effects make it difficult to isolate sources of vibration problems. However, when cavitation occurs, its signature should be contained in the noise floor of the high frequency region due to the unique phenomenon of wide-band modulation. The significance of this phenomenon is that a hydrodynamic source of vibration can now be isolated due to its unique information contained in the high frequency noise floor.

With cavitation detection using the FWRWBD method, the raw high frequency signal must first be high-pass filtered to remove low-frequency discrete components. Demodulation is then performed on the remaining high-frequency noise floor in order to recover any existing low frequency periodic component which is modulating the cavitation generated noise signal. As a result, a new low frequency WBD PSD is generated in addition to the original raw data PSD. However, unlike the raw data PSD which includes multiple contributions from rotordynamics, structure dynamics and hydrodynamics, this new WBD PSD only reflects the hydrodynamic ones. If cavitation does not exist, the WBD PSD should reduce to regular broadband noise. However, if cavitation does occur, the WBD PSD should show discrete peaks corresponding to the low frequency periodic rotational processes modulating collapsing bubble noise

DEMONSTRATION OF THE FWRWBD WITH ENGINE TEST DATA: Real test data from the Space Shuttle Main Engine (SSME) Alternate Turbopump Development (ATD) E8 component test stand is used to demonstrate the effectiveness of the FWRWBD technique in detecting cavitation. Figure 1 shows the ordinary raw PSDs taken from four different accelerometers and pressure measurements across ATD LOX pump unit 3-1A during test E8-162 with a maximum frequency of 5 KHz. The inducer of this unit has four blades. These PSDs all show the fundamental Sync frequency component and its harmonics. Figures 2 shows the ordinary raw PSDs of these same measurements with the maximum frequency increased to 50 KHz. For this engine component test, discrete peaks are concentrated in the low frequency region with only limited amount of high frequency line noise discrete peaks present in the spectra. Notice that the signal energy in the wide-band high frequency noise floor is the prospective information to be used in effective cavitation detection. This high frequency energy may be made up of just regular noise, or may have the very unique wide-band modulation phenomenon hidden within it. Using just the PSDs of both figures 1 and 2, one would not be able to distinguish such a subtle difference.

The high frequency signals shown in figure 2 are first high-pass filtered at 20 KHz to remove all the discrete components, after which demodulation is performed using the FWRWBD method. Figure 3 shows the resulting WBD PSDs of these four measurements. For the first three measurements, which include two accelerometers and one inducer outlet high frequency pressure measurement, no periodic component is recovered in the WBD signal. This indicates that the original wide-band high-frequency noise components of these five measurements are just ordinary noise signals with no modulation phenomenon present. However, in the last plot of figure 3 (figure 3-d), corresponding to the Three Quarter Chord Inducer Inlet Kistler high frequency pressure measurement, several strong discrete components show up in the WBD PSD. This indicates that wide-band modulation phenomenon indeed exists in the high frequency noise floor. In other words, cavitation is present in this test, and the rotational periodic components modulate with collapsing bubble noise generating a unique WBM high-frequency noise floor. The demodulated WBD signal thus recovers these hidden discrete components.

Moreover, an interesting phenomenon is present in this wide-band demodulation result. Notice that this WBD PSD of figure 3 has a strong 2N component and relatively weaker 1N, 3N and 4N components. However, the Raw PSD of the inlet pressure measurement in figure 1 shows a strong 4N component. If a judgement based on this raw PSD had to be made about the cavitation condition, full-bladed (4-blade) cavitation would be the likely candidate. However, the WBD PSD recovers a strong 2N component rather than a 4N component therefore indicating an alternate-blade rather than a full 4-blade cavitation condition, The alternate blade cavitation has been confirmed using high speed video taken during the test for flow visualization. This situation is typical in an actual engine operational environment, since the raw PSD will pick up many other vibration effects in addition to cavitation. The WBD process effectively isolates cavitation signature and provides more reliable information about the true operational condition.

COHERENT PHASE WIDE-BAND DEMODULATION (CPWBD) METHOD: Cavitation detection using the FWRWBD technique is powerful; however it suffers from a commonly encountered limitation. Since the hidden periodicity is recovered from the high frequency noise floor by demodulation, any discrete components present in the high frequency region of the original raw signal will generate false discrete peaks in the WBD signal which appear to be recovered hidden periodicities. These false peaks are generated due to the newly introduced cross coupling components when performing FWRWBD since demodulation represents a nonlinear operation. This limitation may not be so critical in an isolated environment such as during laboratory testing where no discrete components show up in the high frequency region. However, it is a very critical limitation in the analysis of rocket engine static firing or flight data in which many other vibration sources contribute all kinds of high frequency components which corrupt the WBD signal.

Due to its frequency domain formulation of coherence phase information, the Coherent Phase Wide Band Demodulation technique provides an effective method to avoid such discrete interference. As discussed previously, the FWRWBD method is directly related to the special tri-spectrum $T(\omega)$ for hidden periodicity detection in a signal $x(t)$, where $T(\omega)$ is simply equal to the ordinary PSD of the square of $x(t)$. It can be shown that [10], the Discrete Fourier Transform (DFT) of $y(t)=x^2(t)$ can be expressed as:

$$Y(k)=\frac{1}{N}\left[\sum_{i=0}^{N-1} X((i))_N X((k-i))_N\right]R_N(k) \qquad (7)$$

Where X(k) is the DFT of x(t)

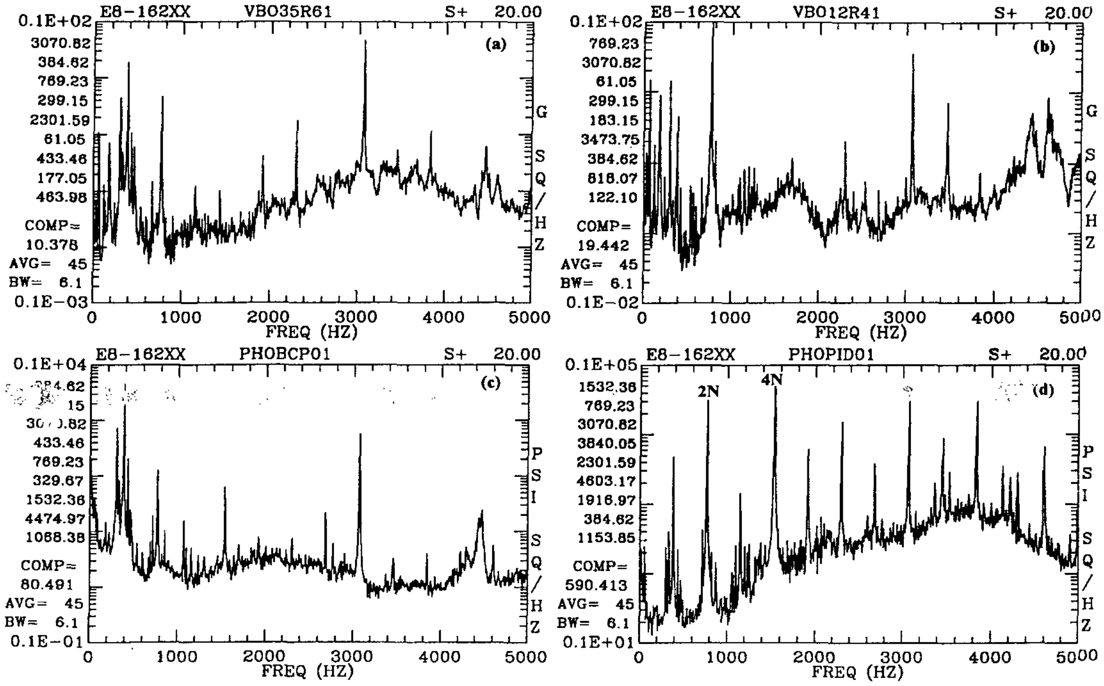

Figure 1: 5 Khz Raw PSDs Taken From Four Different Measurements Across ATD LOX Pump Unit 3-1A During Test E8-162

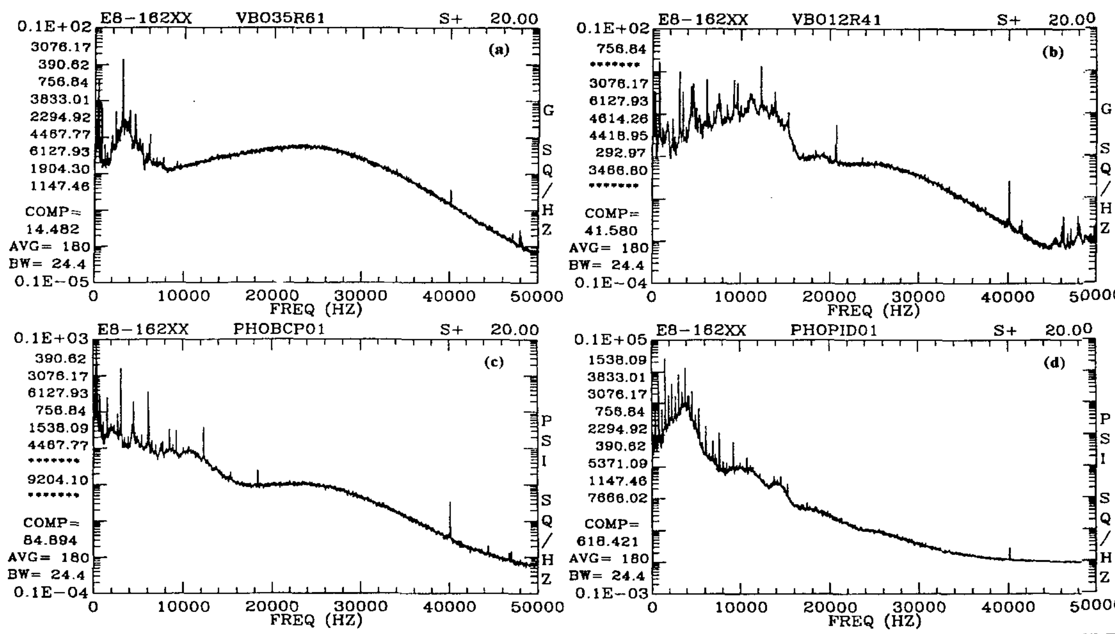

Figure 2: 50 Khz Raw PSDs Taken From Four Different Measurements Across ATD LOX Pump Unit 3-1A During Test E8-162

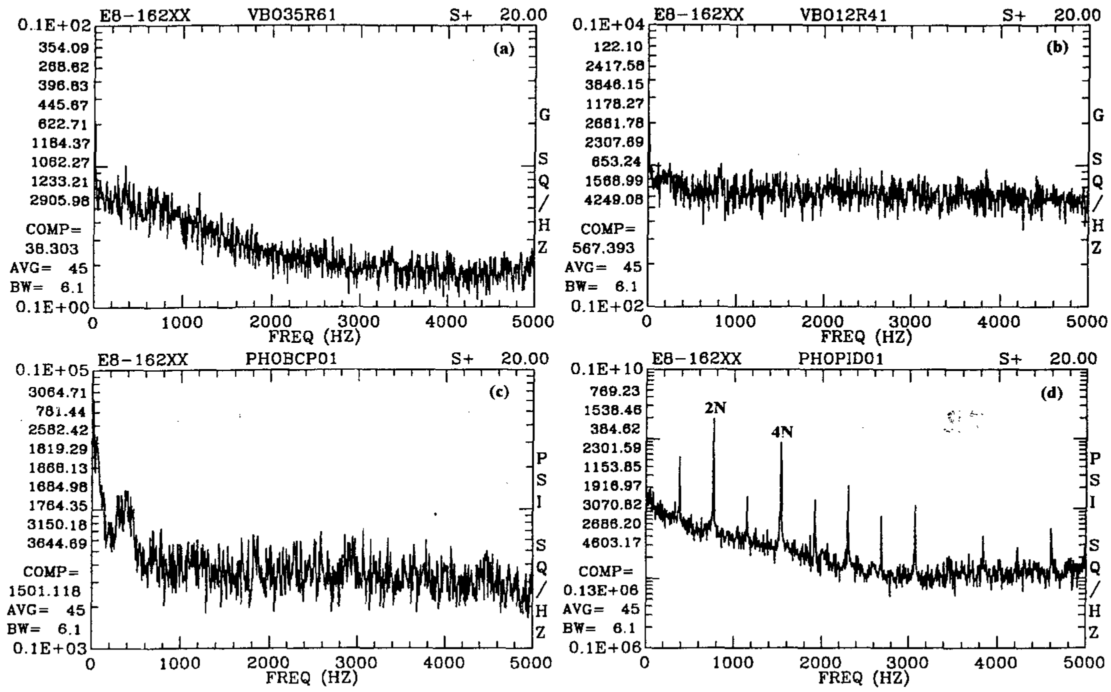

Figure 3: FWRWBD PSDs Obtained From Four Different Measurements In Figure 2

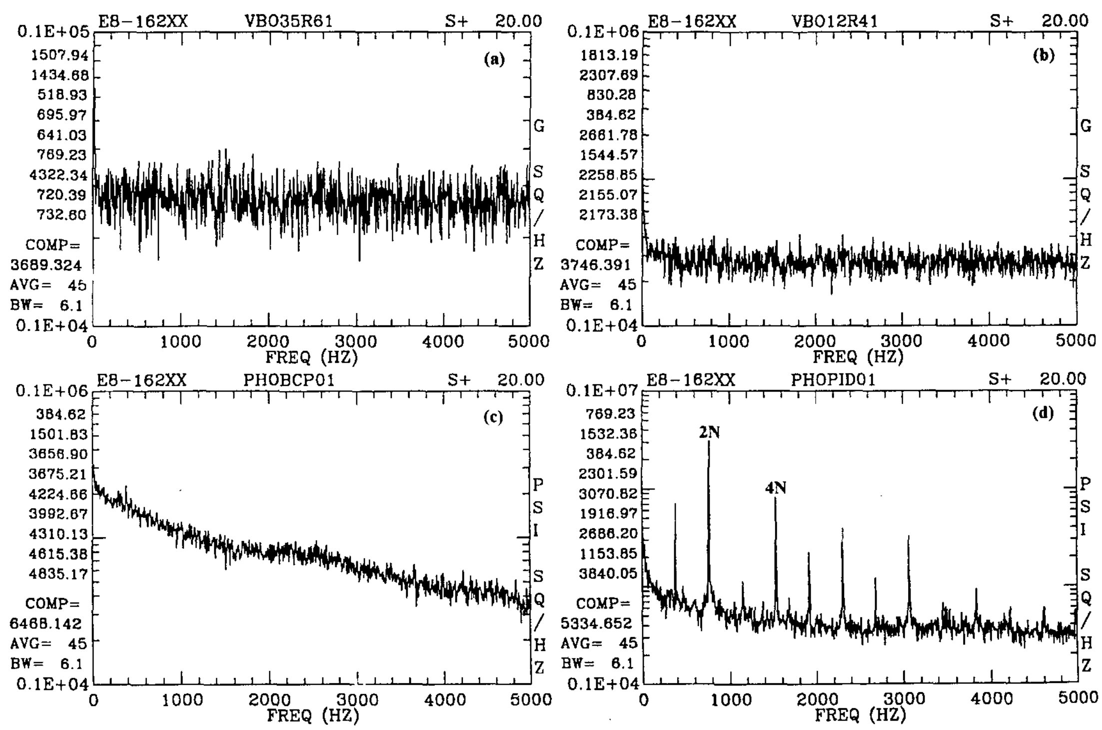

Figure 4: CPWBD PSDs Obtained From Four Different Measurements In Figure 1

$Y(k)$ is the DFT of $y(t)$ (or $x^2(t)$)

$R_N(k)$ is the rectangular function

$((n))_N$ is the (n modulo N) function

In other words, the DFT, $Y(k)$, is equal to the convolution of $X(k)$ with $X(k)$ in the frequency domain. An important observation can be made from this equation; for ordinary independent wide band noise, the phases of $X((i))_N\ X((k-i))_N$ will be independent random phases for i=0 to N-1, therefore, the superposition of all the terms of $X((i))_N\ X((k-i))_N$ during convolution will be a random walk process and vector construction will not occur to form a discrete component in $Y(k)$. On the other hand, if $x(t)$ is a true WBM signal, the coherent phase relationship among all the terms of $X((i))_N\ X((k-i))_N$ will make each individual vector point in a common direction. As a result, the superposition of $X((i))_N\ X((k-i))_N$ during convolution will lead to vector construction, and a hidden periodicity will be recovered in $Y(k)$ at the frequency of the WBM modulating frequency. The significance of this observation is that, the hidden periodicity within a WBM signal can be recovered solely from its unique coherent phase relationship among all interacting components. This unique property thus provides an effective way to solve the discrete interference problem. When an independent discrete component is present within a signal, it will introduce a large step vector in the random walk process during convolution, which would in turn erroneously generate a strong discrete component in the resulting WBD signal pretending to be a recovered hidden periodicity.

To avoid such an unwanted large step random walk due to discrete interference, the CPWBD technique thus performs WBD by introducing a Phase-Only (PO) filter prior to the convolution process. This PO filter will retain the coherent phase relationship associated with a WBM signal to allow hidden periodicity recovery. Furthermore, due to the amplitude normalization effect generated from the PO filter, the CPWBD would no longer be subjected to any independent discrete interference. Due to such a normalization effect, the contribution of unwanted discrete interference is now reduced to the same level of other spectral components. Notice that the amplitude of the hidden periodicity recovered from the CPWBD is also subjected to some normalization. Therefore, the CPWBD should be viewed as a normalized "coherence" function for recovering true hidden periodicity from a WBM signal. It should also be pointed out that, for a noise-free or deterministic signal, both the FWRWBD and CPWBD techniques could produce erroneous results since phase information within the wide band noise floor is heavily corrupted by the side-lobes emanating from the most dominant spectral components, commonly referred to as leakage. Therefore, these techniques should not operate on diagnostic signals with suspiciously "smooth" noise floors corrupted by such leakage.

DEMONSTRATION OF THE FWRWBD WITH ENGINE TEST DATA: The test data from the SSME/ATD E8 component test stand is used to demonstrate the effectiveness of CPWBD method for cavitation detection by only utilizing the low frequency (5 KHz) signal. The objective here is to demonstrate that (1) if the WBM phenomenon also exists at lower frequencies, the hidden periodicity associated with cavitation can also be recovered from the low frequency (5 Khz) signal rather than having to perform high frequency analysis which requires more processing time and larger data volume; (2) the apparent discrete components (e.g. Sync and Harmonics) in the low frequency region will not affect the CPWBD technique in detecting hidden periodicity (Notice that, a direct application of FWRWBD to the 5 KHz frequency signal will unavoidably introduce erroneous peaks during the demodulation process due to these apparent discrete components).

Wide-band demodulation is performed on the 5 KHz frequency data by the CPWBD method with its resulting WBD PSDs shown in figures 4. The WBD PSD in figure 4-d clearly shows that strong hidden periodicity at 2N due to alternate blade cavitation is identified. The more significant result is that, erroneous peaks are no longer present in the other three measurements in figures 4-a, 4-b, and 4-c, as would be with the FWRWBD method. This is because the CPWBD technique effectively identifies hidden periodicity by searching for a coherent phase relationship and is not be affected by linearly superimposed discrete components. The original discrete components, unrelated to cavitation, in the raw PSDs are therefore rejected during the demodulation due to the lack of phase correlation.

CONCLUSION: Cavitation occurs in many types of turbomachinery, causing performance degradation, excessive structural vibration, material erosion and pitting damage. The principle for cavitation detection is based on the nonlinear modulation phenomenon between the bubble collapse generated wide-band noise with some synchronous (RPM or harmonics) and/or non-synchronous periodic components. However, the periodicities modulating the wide band noise become hidden in the resulting WBM signal and cannot be identified with the conventional power spectral density function. Such hidden periodicity thus provides a unique signature in the wide band noise conducive to cavitation detection. When a measured diagnostic signal is not subjected to discrete components interference due to other vibration sources, the full-wave rectification technique is an effective method to detect the existence and identify the intensity of cavitation.

The CPWBD method utilizes a Phase-Only filter coupled with envelope analysis to search for a unique coherent phase relationship associated with the cavitation-generated wideband modulation phenomenon. As a result, the cavitation analysis/prediction limitation due to discrete interference is eliminated. Analysis results from actual SSME ATD engine test data indicate that the CPWBD can effectively detect cavitation signatures by searching for such coherent phase relationships. In addition, the signal processing associated with the CPWBD can take advantage of the computational efficiency of Fast Fourier Transform (FFT) making it highly suitable for real-time implementation in an on-line cavitation monitoring system.

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